MATH152 CALCULUS II TUTORIAL – I

(13.03.2015)

Question 1 : (Geometric Series)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{4}{2^n}$$

1. $\sum_{n=0}^{\infty} \frac{4}{2^n} = 4\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n$
2. Geometric series with $r = \frac{1}{2}$
3. Converges by Theorem 9.6

Question 2: (Geometric Series)

Find the sum of the convergent series.

$$\sum_{n=0}^{\infty}\left(\frac{1}{2^n} - \frac{1}{3^n}\right)$$

1. $\sum_{n=0}^{\infty}\left(\frac{1}{2^n} - \frac{1}{3^n}\right) = \sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^n - \sum_{n=0}^{\infty}\left(\frac{1}{3}\right)^n$
2. $= \frac{1}{1-(1/2)} - \frac{1}{1-(1/3)}$
3. $= 2 - \frac{3}{2}$
4. $= \frac{1}{2}$

Question 3: (Geometric Series)

Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-2}}{2^n}$$

1. $\sum_{n=1}^{\infty} \frac{(-1)^n 3^{n-2}}{2^n} = \sum_{n=1}^{\infty} \frac{(-1)^n 3^n 3^{-2}}{2^n}$
2. $= \sum_{n=1}^{\infty} \frac{1}{9}\left(-\frac{3}{2}\right)^n$
3. Since $|r| = \frac{3}{2} > 1$, this is a divergent geometric series.

Question 4: (n'th term test)

Verify that the infinite series diverges.

$$\sum_{n=1}^{\infty} \frac{n^2}{n^2+1}$$

1. $\lim_{n\to\infty} \frac{n^2}{n^2+1} = 1$
2. $\neq 0$
3. Diverges by Theorem 9.9

Question 5: (n'th term test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{3n-1}{2n+1}$$

1. $\lim_{n\to\infty} \frac{3n-1}{2n+1} = \frac{3}{2}$
2. $\neq 0$
3. Diverges by Theorem 9.9

Question : 6 (p-test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}}$$

1. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt[4]{n}} = \sum_{n=1}^{\infty} \frac{1}{n^{5/4}}$
2. p-series with $p = \frac{5}{4}$
3. Converges by Theorem 9.11

Question : 7 (Comparision test)

Use the Direct Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{1}{3^n + 1}$$

1. $0 < \frac{1}{3^n + 1} < \frac{1}{3^n}$
2. Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{3^n + 1}$$

converges by comparison with the convergent geometric series

$$\sum_{n=0}^{\infty} \left(\frac{1}{3}\right)^n.$$

Question 10 (Ratio test)

Determine the convergence or divergence of the series using any appropriate test from this chapter. Identify the test used.

$$\sum_{n=1}^{\infty} \frac{n7^n}{n!}$$

1. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)7^{n+1}}{(n+1)!} \cdot \frac{n!}{n7^n}\right|$
2. $= \lim_{n\to\infty} \frac{7}{n}$
3. $= 0$
4. Therefore, by the Ratio Test, the series converges.

Question 8 (Comparision test)

Use the Direct Comparison Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

1. For $n > 3$,
2. $\frac{1}{n^2} > \frac{1}{n!} > 0.$
3. Therefore,

$$\sum_{n=0}^{\infty} \frac{1}{n!}$$

converges by comparison with the convergent p-series

$$\sum_{n=1}^{\infty} \frac{1}{n^2}.$$

Question 11 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

1. $a_{n+1} = \frac{1}{\sqrt{n+1}}$
2. $< \frac{1}{\sqrt{n}}$
3. $= a_n$
4. $\lim_{n\to\infty} \frac{1}{\sqrt{n}} = 0$
5. Converges by Theorem 9.14

Question 9 (Ratio Test)

Use the Ratio Test to determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{n!}{3^n}$$

1. $\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!}{3^{n+1}} \cdot \frac{3^n}{n!}\right|$
2. $= \lim_{n\to\infty} \frac{n+1}{3}$
3. $= \infty$
4. Therefore, by the Ratio Test, the series diverges.

Question 12 (Alternating series Test)

Determine the convergence or divergence of the series.

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

1. $a_{n+1} = \frac{1}{(n+1)!}$
2. $< \frac{1}{n!}$
3. $= a_n$
4. $\lim_{n\to\infty} \frac{1}{n!} = 0$
5. Converges by Theorem 9.14

Question 13

Find the Maclaurin polynomial of degree n for the function.

$f(x) = e^{-x}, \quad n = 3$

1. $f(x) = e^{-x}$
2. $f(0) = 1$
3. $f'(x) = -e^{-x}$
4. $f'(0) = -1$
5. $f''(x) = e^{-x}$
6. $f''(0) = 1$
7. $f'''(x) = -e^{-x}$
8. $f'''(0) = -1$
9. $P_3(x) = f(0) + f'(0)x + \dfrac{f''(0)}{2!}x^2 + \dfrac{f'''(0)}{3!}x^3$
10. $\quad = 1 - x + \dfrac{x^2}{2} - \dfrac{x^3}{6}$

Question 14

Find the Maclaurin polynomial of degree n for the function.

$f(x) = \dfrac{1}{x+1}, \quad n = 4$

1. $f(x) = \dfrac{1}{x+1}$
2. $f(0) = 1$
3. $f'(x) = -\dfrac{1}{(x+1)^2}$
4. $f'(0) = -1$
5. $f''(x) = \dfrac{2}{(x+1)^3}$
6. $f''(0) = 2$
7. $f'''(x) = \dfrac{-6}{(x+1)^4}$
8. $f'''(0) = -6$
9. $f^{(4)}(x) = \dfrac{24}{(x+1)^5}$
10. $f^{(4)}(0) = 24$
11. $P_4(x) = 1 - x + \dfrac{2}{2!}x^2 + \dfrac{-6}{3!}x^3 + \dfrac{24}{4!}x^4$
12. $\quad = 1 - x + x^2 - x^3 + x^4$

Question 15

Find the values of x for which the series converges.

$\sum_{n=0}^{\infty} 2\left(\dfrac{x}{3}\right)^n$

1. $\lim_{n\to\infty}\left|\dfrac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\left|\dfrac{2(x/3)^{n+1}}{2(x/3)^n}\right|$
2. $\quad = \lim_{n\to\infty}\left|\dfrac{x}{3}\right|$
3. $\quad = \left|\dfrac{x}{3}\right|$
4. For the series to converge: $\left|\dfrac{x}{3}\right| < 1$
5. $\quad \Rightarrow -3 < x < 3.$
6. For $x = 3$, the series diverges.
7. For $x = -3$, the series diverges.
8. Answer: $-3 < x < 3$

Question 16

Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$\sum_{n=1}^{\infty} \dfrac{(x-3)^{n-1}}{3^{n-1}}$

1. $\sum_{n=1}^{\infty}\left(\dfrac{x-3}{3}\right)^{n-1}$ is geometric.
2. It converges if $\left|\dfrac{x-3}{3}\right| < 1$
3. $\quad \Rightarrow |x-3| < 3$
4. $\quad \Rightarrow 0 < x < 6.$
5. Interval convergence: $0 < x < 6$

Question 17

Approximate the function at the given value of x, using the polynomial found in the indicated exercise

$f(x) = \ln x, f(1.2),$

Exercise 29

1. $f(x) = \ln x$
2. $\approx (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$
3. $f(1.2) \approx 0.1823$

Question 18

Find the values of x for which the series converges.

$$\sum_{n=0}^{\infty} n!\left(\frac{x}{2}\right)^n$$

1. $\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty}\frac{(n+1)!|x/2|^{n+1}}{n!|x/2|^n}$
2. $= \lim_{n\to\infty}(n+1)\left|\frac{x}{2}\right|$
3. $= \infty$
4. The series converges only at $x = 0$.

Question 19

Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\sum_{n=0}^{\infty} n!(x-2)^n$$

1. $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(n+1)!(x-2)^{n+1}}{n!(x-2)^n}\right|$
2. $= \infty$
3. which implies that the series converges only at the center $x = 2$.

Question 20

Find the interval of convergence of the power series. (Be sure to include a check for convergence at the endpoints of the interval.)

$$\sum_{n=0}^{\infty}\frac{(-1)^n(x-2)^n}{(n+1)^2}$$

1. $\lim_{n\to\infty}\left|\frac{u_{n+1}}{u_n}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}(x-2)^{n+1}}{(n+2)^2}\cdot\frac{(n+1)^2}{(-1)^n(x-2)^n}\right|$
2. $= |x-2|$
3. $R = 1$
4. Center: 2
5. Since the series converges when $x = 1$ and when $x = 3$,
6. the interval of convergence is $1 \le x \le 3$.